# Hydrodynamics of Large Objects in the Sea Part II: Motion of Free-Floating Bodies

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**A numerical scheme was developed in Part I utilizing digital computer calculations to determine wave excitation forces as well as added mass and damping coefficients for large objects in the sea. The analysis was carried out within the framework of linear theory for bodies of arbitrary shape, either submerged or semisubmerged, in water of finite depth. In this Part II, an analysis is presented for the motion of large free-floating bodies. The results obtained by methods of Part I are utilized in conjunction with the equations of motion of the floating body to determine the response induced by wave excitation. Numerical results are presented for the response of a sphere and a short, vertical, circular cylinder floating in water of finite depth.**

## Nomenclature

$\bar{a}$ = characteristic dimension of the body
$C_j$ = $F_j(max)e^{i\delta_j}/\rho g\bar{a}^3\eta^\circ$ = wave excitation force coefficient ($j=1,2,3$)
$C_j$ = $F_j(max)e^{i\delta_j}/\rho g\bar{a}^4\eta^\circ$ = wave excitation moment coefficient ($j=4,5,6$)
$c_j(t)$ = $F_j(t)/\rho g\bar{a}^3\eta^\circ$, ($j=1,2,3$), wave excitation force coefficient.
$c_j(t)$ = $F_j(t)/\rho g\bar{a}^4\eta^\circ$, ($j=4,5,6$), wave excitation moment coefficient.
$c_j(t)$ = $Re[C_i e^{-i\sigma t}[=|C_i|Re[e^{i(\delta_i-\sigma t)}]$
$c_{ij}(t)$ = $F_{ij}(t)/\rho g\bar{a}^3\eta^\circ$, ($j=1,2,3$), force coefficient, see Eq. (9)
$c_{ij}(t)$ = $F_{ij}(t)/\rho g\bar{a}^4\eta^\circ$, ($i=4,5,6$), moment coefficient, see Eq. (9)
$\bar{d}$ = depth of submergence of body.
$f_i^T(t)$ = total force or moment coefficient, see Eq. (14)
$F_{ij}(t)$ = force in $i$-direction due to oscillation in $j$-mode ($i=1,2,3; j=1,2,\ldots 6$)
$F_{ij}(t)$ = moment about $i$-axis due to oscillation in $j$-mode ($i=4,5,6; j=1,2,\ldots 6$)
$F_i^H$ = force in $i$-direction due to hydrostatic pressure ($i=1,2,3$)
$F_{i+3}^H$ = moment about $i$-axis due to hydrostatic pressure
$F_{ij}^H$ = $i$-component of force (or moment) due to $j$-component of displacement, see Eq. (13)
$F_i^T$ = components of the total force (or moment) vector
$g$ = acceleration of gravity
$g_i$ = see Eq. (11)
$h$ = water depth
$\bar{H}$ = wave height (crest to trough)
$\bar{I}$ = mass moment of inertia, see Eq. (3)
$K_{ij}$ = see Eq. (14) and Eqs. (16-21)
$k_{ij}(t)$ = $F_{ij}^H/\rho g\bar{a}^3$, ($i=1,2,3$) hydrostatic force coefficient
$k_{ij}(t)$ = $F_{ij}^H/\rho g\bar{a}^4$, ($i=4,5,6$) hydrostatic moment coefficient
$\bar{m}$ = mass of body (displaced mass)
$m_{ij}$ = dimensionless moment of inertia, see Eq. (4) ($i \neq j$)
$m_{ii}$ = body mass/$\rho\bar{a}^3$
$M_{ij}$ = added mass or moment of inertia coefficients
$N_{ij}$ = damping coefficients
$S$ = surface area of the body
$T$ = wave period
$t$ = time
$\bar{X}$ = surge, heave and sway displacements, respectively
$X_i$ = dimensionless displacements, ($i=1,2,\ldots 6$)
$X_i^\circ$ = dimensionless displacement amplitudes, ($i=1,2,\ldots 6$)
$\delta_i$ = phase shift of wave excitation force or moment (reference condition is where undisturbed incident wave is centered over body)
$\eta^\circ$ = $\bar{H}/2\bar{a}$
$\theta_4,\theta_5,\theta_6$ = roll, yaw and pitch angles, respectively
$\nu$ = $\sigma^2\bar{a}/g$
$\rho$ = fluid density
$\sigma$ = $2\pi/T$
$\psi_i$ = phase shift of the $i$-mode of body motion, see Eq. (23)

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## Introduction

IN the past, the primary application of research relevant to the wave induced response of floating bodies has been to surface ships. In view of the elongated shape of typical ships, a two-dimensional hydrodynamic analysis (strip theory) is generally employed. The assumption of infinite depth is also common to most of the published work in ship hydrodynamics. However, currently large caissons and gravity structures are being proposed for ocean deployment. Such proposals usually involve towing the caisson or structure to the deployment site and then sinking it in a controlled manner. The towing and sinking operations involve floating of large three-dimensional objects in water of finite depth and, consequently, the response of such structures to wave motion is of concern. Ocean structures are generally not constructed in the form of elongated bodies as in the case of a ship and, therefore, strip theory is invalid, and a truly three-dimensional hydrodynamic analysis is required. Moreover, during the sinking operation the large object approaches and eventually sits on the ocean floor, and consequently, the bottom proximity or finite depth effect is also of particular interest. These effects have received very little attention by naval architects in the past.

Several papers have treated the hydrodynamic coefficients for the two-dimensional problem associated with cylinders oscillating in water of infinite depth. Examples include those of Ursell,[2] Porter,[3] Vugts,[4] and Paulling and Richardson.[5] Equivalent data for three-dimensional shapes is much more limited, but Havelock[6] has theoretically determined added mass and damping coefficients for a floating sphere, and ellipsoidal bodies oscillating on the free surface have been considered by Kim.[7] In a later paper Kim[8] calculated the heave, surge, and pitch response for the same spheroidal bodies to wave excitation. Barakat[9] also treated the vertical motion of a sphere as induced by surface waves, but his work unfortunately contained errors. Vugts[10] has computed the heaving, swaying, and rolling response of a rectangular cylinder with rounded corners.

In most of the literature on the hydrodynamics of ships only the added mass and damping coefficients are determined. However, in order to determine body motion, it is necessary to know the wave excitation forces and moments; and if the equations of motion are coupled, which is the most general case, it is necessary to know, in addition, the phase relationships between the components of the forces and moments. In most of the literature the coupling coefficients of added mass and damping have been disregarded because they are generally not of major importance in the case of ships.

In this paper, the equations of motion for a free-floating body are developed. These equations are then applied using hydrodynamic coefficients obtained by methods described in Part I[1] to determine actual motion response of certain floating bodies.

## Equations of Motion

The problem under consideration is represented schematically in Fig. 1. A small amplitude regular wave train of wave height $\bar{H}$ is considered to progress in water of depth $\bar{h}$ and interact with the floating body. The mean position of the center of mass of the body is located a distance $\bar{d}$ beneath the mean free surface and a coordinate system $\bar{x}', \bar{y}', \bar{z}'$ isattached to the body with origin at the mass center. The body coordinate system in its mean position is set parallel to the inertial coordinate system with its origin located at $(0,-\bar{d},0)$. The small amplitude displacement of the body center of mass with respect to its mean position in the inertial reference frame is described by the three coordinates $\bar{X}_1(t), \bar{X}_2(t)$, and $\bar{X}_3(t)$ which are referred to as surge, heave, and sway, respectively. The small angular displacements of the body about the $\bar{x}'$, $\bar{y}'$, and $\bar{z}'$ axes are denoted by $\theta_4$, $\theta_5$, and $\theta_6$ and are referred to as roll, yaw and pitch, respectively.

The equations of motion linearized with respect to the small angular displacements of the body may now be written as follows:

$$F_1{}^T(t)=\bar{m}\ddot{\bar{X}}_1(t)$$
$$F_2{}^T(t)=\bar{m}\ddot{\bar{X}}_2(t) \qquad (1)$$
$$F_3{}^T(t)=\bar{m}\ddot{\bar{X}}_3(t)$$

$$F_4{}^T(t)=\bar{I}_{44}\ddot{\theta}_4-\bar{I}_{45}\ddot{\theta}_5-\bar{I}_{46}\ddot{\theta}_6$$
$$F_5{}^T(t)=\bar{I}_{55}\ddot{\theta}_5-\bar{I}_{56}\ddot{\theta}_6-\bar{I}_{54}\ddot{\theta}_4 \qquad (2)$$
$$F_6{}^T(t)=\bar{I}_{66}\ddot{\theta}_6-\bar{I}_{64}\ddot{\theta}_4-\bar{I}_{65}\ddot{\theta}_5$$

where $F_1{}^T(t)$, $F_2{}^T(t)$, and $F_3{}^T(t)$ denote the three components of the total external force acting on the body and $F_4{}^T(t)$, $F_5{}^T(t)$, and $F_6{}^T(t)$ denote the three components of the total external moment. The symbol $\bar{m}$ denotes the body mass which equals the displaced mass. The moments of inertia are defined, typically, as

$$\bar{I}_{45}=\bar{I}_{54}=\int_{\bar{m}}\bar{x}'\,\bar{y}'\,d\bar{m} \qquad (3)$$

where the integration is to be carried out over the complete mass of the body. For bodies having symmetry with respect to the $(\bar{x}'-\bar{y}')$ and $(\bar{y}'-\bar{z}')$ planes, all of the products of inertia vanish. Although this type of symmetry is common to most ocean structures, there is no need to apply the limitation at this point since the inclusion of the product of inertia terms do not, in principle, complicate the development.

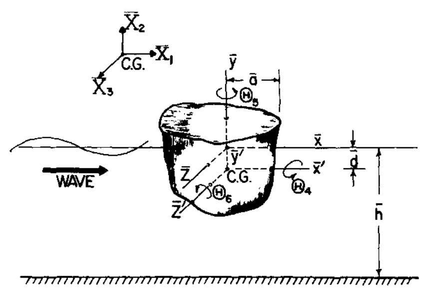


**Fig. 1 Definitions.**

At this juncture it is convenient to place the equations of motion in dimensionless form. We may define the following dimensionless parameters for this purpose:

$$m_{11}=m_{22}=m_{33}=\bar{m}/\rho\bar{a}^3,$$
$$m_{12}=m_{21}=m_{13}=m_{31}=m_{23}=m_{32}=0$$
$$m_{44}=\bar{I}_{44}/\rho\bar{a}^5,\ m_{55}=\bar{I}_{55}/\rho\bar{a}^5, m_{66}=\bar{I}_{66}/\rho\bar{a}^5$$
$$m_{45}=m_{54}=-\bar{I}_{54}/\rho\bar{a}^5, m_{46}=m_{64}=-\bar{I}_{46}/\rho\bar{a}^5$$
$$m_{56}=m_{65}=-\bar{I}_{56}/\rho\bar{a}^5 \qquad (4)$$
$$X_i(t)=\bar{X}_i(t)/\bar{a},\ i=1,2,3$$
$$X_i(t)=\theta_i(t),\ i=4,5,6$$
$$f_i^T(t)=F_i^T(t)/\rho g\bar{a}^3,\ i=1,2,3$$
$$f_i^T(t)=F_i^T(t)/\rho g\bar{a}^4,\ i=4,5,6$$

where $\bar{a}$ denotes the characteristic dimension of the body, $\rho$ denotes the fluid density and $g$ denotes the gravitational constant.

Using these definitions, Eqs. (1) and (2) may be condensed to the form:

$$f_i^T(t)=(\bar{a}/g)m_{ij}\ddot{X}_j(t) \qquad (5)$$

where $i$ takes on values 1,2,...6 and the repeated index denotes summation as usual.

Equation (5) represents the six equations of motion with $f_i^T(t)$ denoting the dimensionless total external force or moment coefficients as defined by Eq. (4). For free-floating bodies these coefficients represent the contributions from the surrounding fluid only and are generally considered to be composed of three parts( a) the wave excitation forces and moments; b) the dynamic forces and moments caused by the responsive motion of the body, and c) the hydrostatic forces and moments caused by body displacements. For the linear problem these three contributions may be determined separately and superimposed.

In accordance with this idea the three contributions to the total force (or moments) coefficient $f_i^T(t)$ may be expressed as the linear combination:

$$f_i^T(t)=C_i(t)+\sum_{j=1}^{6}[C_{ij}(t)+k_{ij}(t)] \qquad (6)$$

where $c_i(t)$ = force or moment coefficient associated with wave excitation, $F_i(t)/\rho g\bar{a}^3, i=1,2,3$; $F_i(t)/\rho g\bar{a}^4, i=4,5,6$; $c_{ij}(t)$ = force or moment coefficient associated with the dynamic response of the body; and $k_{ij}(t)$ = force or moment coefficient associated with linear or angular displacement resulting from hydrostatic pressures.

The force or moment coefficient $c_i(t)$ may be expressed as

$$c_i(t)=\eta^\circ Re[|C_i|e^{i\delta_i}e^{i\sigma t}], i=1,2,...6 \quad (7)$$

where $i=1,2,3$ refers to the three components of the force and $i=4,5,6$ refers to the three components of the moment. The dimensionless coefficient $c_i(t)$ is defined according to the definition of $f_i^T(t)$ as the force made dimensionless with $\rho g\bar{a}^3$ $(i=1,2,3)$ or the moment made dimensionless with $\rho g\bar{a}^4$ $(i=4,5,6)$. The frequency of the wave excitation is denoted by $\sigma$ and $\delta_i$ denotes the phase shift angle of the $i^{th}$ component of force or moment in relationship to the incident wave. In Part I,[1] as well as in the present paper, all phase shift angles are measured relative to the incident wave with the $t=0$ condition corresponding to the condition when the crest of the undisturbed incident wave is at the corrdinate origin. The magnitude of the complex force or moment coefficient occurring in Eq. (7) is defined, as in Part I, as

$$|C_i|=\frac{F_i(max)}{\rho g\bar{a}^3\eta^\circ}, \ i=1,2,3;$$

$$|C_i|=\frac{F_i(max)}{\rho g\bar{a}^4\eta^\circ}, \ i=4,5,6 \quad (8)$$

where $\eta^\circ=\bar{H}/2\bar{a}$ denotes the dimensionless wave amplitude, $\bar{H}$ being the wave height and $\bar{a}$ the characteristic body dimension. The amplitudes of the excitation forces and moments are denoted by $F_{i(max)}$ where $i=1,2,3$ refers to the three components of force and $i=4,5,6$ refers to the moments.

As the body responds to the wave excitation, dynamic pressures arise due to the motion which may be resolved into two components of force (or moment), one in phase and proportional to the acceleration of the body and a second in phase and proportional to the velocity of the body. These two components are characterized by dimensionless added mass coefficients, $M_{ij}$, and damping coefficients, $N_{ij}$. According to the definitions of these parameters, $c_{ij}(t)$ which denotes the $i$-component force due to the $j$-component motion made dimensionless with $\rho g\bar{a}^3$ $(i=1,2,3)$ or the corresponding moment made dimensionless with $\rho g\bar{a}^4$ $(i=4,5,6)$, is given by

$$c_{ij}(t)=-(\bar{a}/g)M_{ij}\ddot{X}_j(t)-(\bar{a}\sigma/g)N_{ij}\dot{X}_j(t) \quad (9)$$

where $j=1,2,...,6$ denotes the six degrees of freedom. The added mass and damping coefficients are dependent on the shape of the body, the water depth, and the frequency parameter, $\nu=\sigma^2\bar{a}/g$. The evaluation of these coefficients was dealt with in Part I.

The final contribution to the force (or moment) resulting from the surrounding fluid comes from the hydrostatic pressure. As the body is displaced from its equilibrium position, forces and moments arise which are proportional to the body displacement. The hydrostatic pressure increases with depth according to $p=-\rho g\bar{y}$ and, consequently, the $i^{th}$ component of hydrostatic force or moment resulting from this pressure variation and acting on the body is given by the integrals

$$F_i{}^H=\rho g\bar{a}^3\iint_s y\,g_i\,dS, i=1,2,3;$$

$$F_i{}^H=\rho g\bar{a}^4\iint_s y\,g_i ds, i=4,5,6 \quad (10)$$

where $y=\bar{y}/\bar{a}$ denotes the dimensionless $y$ coordinate of a point on the immersed surface, and $dS=s\bar{S}/\bar{a}^2, d\bar{S}$ being an elemental surface area. The functions $g_i$ occurring in Eq. (10) are defined as follows:

$$g_1=n_x,\ g_2=n_y,\ g_3=n_z$$

$$g_4=y'n_z-z'n_y,\ g_5=z'n_x-x'n,\ g_6=x'n_y-y'n_x \quad (11)$$

where the dimensionless body coordinates are defined as

$$x'=\bar{x}'/\bar{a},\ y'=\bar{y}/\bar{a},\ z'=\bar{z}'/\bar{a}, \text{ and } d=\bar{d}/\bar{a}$$

and the unit normal vector on the immersed surface is defined as $\vec{n}(x', y', z'; X_1, X_2, X_3)=\vec{i}n_y+\vec{j}n_y+\vec{n}_z$. Moreover, the following linearized onship exist between the coordinates of the fixed reference frame and the body coordinates for a given point on the body surface located at $x'$, $y'$, $z'$:

$$x=x'+X_5z'-X_6y'+X_1$$

$$y=-d+y'+X_6x'-X_4z'+X_2 \quad (12)$$

$$z=z'+X_4y'-X_5x'+X_3$$

Equation (10) represents the hydrostatic force (or moment) acting on the immersed surface including the buoyant force which must, of course, just balance the weight. It is desired, however, to determine the $i^{th}$ component of force associated with a $j^{th}$ $(j=1,2,...6)$ component of displacement of the body, $j=1,2,3$ denoting displacement in the $x,y,z$ directions, respectively, and $j=4,5,6$ denoting angular displacement about the $x',y',z'$ axes, respectively.

For the linearized problem this force (or moment) may be written, using Eq. (10), as:

$$F_{ij}^H=\frac{\partial F_i{}^H}{\partial X_j}X_j=(1 \text{ or } \bar{a})\rho g\bar{a}^3X_j\frac{\partial}{\partial X_j}\iint_s y\,g_i ds \quad (13)$$

[The factor 1.0 in brackets in Eq. (13) is applied when $i=1,2,3$ and $\bar{a}$ is appropriate when $F_{ij}$ denotes a moment, i.e., $i=4,5,6$.] Defining, further, the dimensionless parameter $K_{ij}$ as

$$K_{ij}=-\iint_s\frac{\partial}{\partial X_j}(y\,g_i)ds \quad (14)$$

we may write the dimensionless force (or moment) coefficient $k_{ij}(t)$, in view of Eq. (13), as

$$k_{ij}(t)=K_{ij}X_j(t) \text{ (no sum)} \quad (15)$$

Equation (15) represents a form appropriate to Eq. (16) for the dimensionless force in the $i^{th}$ direction caused by a displacement in the $j^{th}$ degree of freedom. It remains to evaluate the expressions for the coefficients, $K_{ij}$.

Within the framework of linear analysis, Table 1 may be constructed as shown. Then, using these results to evaluate the derivatives indicated in Eq. (14), where $g_i$ is given by Eq. (11), the following expressions are obtained for the coefficients $K_{ij}$:

$$K_{22}=-\iint_s n_y ds=A_w/\bar{a}^2 \quad (16)$$

$$K_{24}=K_{42}=\iint_s z'n_y dS \quad (17)$$

**Table 1 Derivatives**

| | $x$ | $y$ | $z$ | $n_x$ | $n_y$ | $n_z$ |
|---|---|---|---|---|---|---|
| $\partial/\partial X_1$ | 1 | 0 | 0 | 0 | 0 | 0 |
| $\partial/\partial X_2$ | 0 | 1 | 0 | 0 | 0 | 0 |
| $\partial/\partial X_3$ | 0 | 0 | 1 | 0 | 0 | 0 |
| $\partial/\partial X_4$ | 0 | $-z'$ | $y'$ | 0 | $-n_z$ | $n_y$ |
| $\partial/\partial X_5$ | $z'$ | 0 | $-x'$ | $n_z$ | 0 | $-n_x$ |
| $\partial/\partial X_6$ | $-y'$ | $x'$ | 0 | $-n_y$ | $n_x$ | 0 |

$$K_{44}=\iint_s(y'z'n_z-z'^2n_y)dS \quad (18)$$

$$K_{46}=K_{64}=\iint_s x'z'n_y dS \quad (19)$$

$$K_{26}=K_{62}=-\iint_s x'n_y dS \quad (20)$$

$$K_{66}=\iint_s(x'y'n_x-x'^2n_y)dS \quad (21)$$

in which $A_w$ denotes the waterplane area in the equilibrium position. For floating bodies having symmetry with respect to the $x'-y'$ and $y'-z'$ planes the coefficients, $K_{24}=K_{42}=K_{46}=K_{64}=K_{26}=K_{62}=0$.

It is noted here that the expressions, Eqs. (16-21), for the hydrostatic force coefficients are in rather convenient forms for evaluation as a part of the same computer program used for the evaluation of the hydrodynamic coefficients as discussed in Part I.[1] The numerical approach in Part I involved representation of the immersed surface by a large number of nodal points. At each point it was necessary to specify the three coordinates of its location, the three components of the unit vector normal to the surface, and the elemental area of the facet. This information is, therefore, available for the evaluation of Eqs. (16-21).

The equations of motion for a free floating body may now be obtained by use of Eqs. (5-7, 9, and 15) as follows:

$$\frac{\bar{a}}{g}(m_{ij}+M_{ij})\ddot{X}_j+N_{ij}\frac{\bar{a}\sigma}{g}\dot{X}_j+K_{ij}X_j$$
$$=\eta^\circ Re[|C_i|e^{i\delta_i}e^{-i\sigma t}] \quad (22)$$

Furthermore, the body response in the $j^{th}$ degree of freedom may be expressed in the form

$$X_j(t)=X_j^\circ Re[e^{i\psi_j}e^{-i\sigma t}] \quad (23)$$

which, when substituted into Eq. (22), yields the complex equations of motion,

$$\left[-(m_{ij}+M_{ij})-iN_{ij}+\frac{K_{ij}}{\nu}\right]\frac{X_j^\circ}{\eta^\circ}e^{i\psi_j}=\frac{|C_i|}{\nu}e^{i\delta_i} \quad (24)$$

where the frequency parameter, $\nu=\sigma^2\bar{a}/g$. Equation (24) represents six equations corresponding to $i=1,2,...6$. The repeated $j$ index denotes as usual the summation over the six degrees of freedom. The amplitude ratio $X_j^\circ/\eta^\circ$ denotes the ratio of the amplitude of the motion to the amplitude of the incident wave and $\psi_j$ denotes the phase angle of the motion in relation to the reference condition of the crest of the incident wave being located at the coordinate origin.

It will be recalled that for bodies possessing $x'-y'$ and $y'-z'$ plane symmetry, $m_{ij}=0$ for $i\neq j$ in Eq. (24). Also, $M_{ij}$ and $N_{ij}$ denote the thirty-six element added mass and damping tensors. It can be shown, however, that

$$K_{ij}=K_{ji},\ M_{ij}=M_{ji},\ N_{ij}=N_{ji} \quad (25)$$

To further simplify the notation, we may define the complex matrix,

$$A_{ij}=-(m_{ij}+M_{ij})+K_{ij}/\nu-i\,N_{ij} \quad (26)$$

and the complex vectors:

$$\alpha_j=\frac{X_j^\circ}{\eta^\circ}e^{i\psi_j} \quad (27)$$

and

$$B_i=\frac{|C_i|}{\nu}e^{i\delta_i} \quad (28)$$

Then, Eq. (24) may be written simply as

$$A_{ij}\alpha_j=B_i,\ i,j=1,2,...6 \quad (29)$$

Once the solution to Eq. (29) for $\alpha_j$ is obtained, the problem is solved. The amplitude and phase angle of the response can be extracted from the definition of $\alpha_j$ given in Eq. (27).

## Numerical Results

Wave excitation forces and moments, and added mass and damping coefficients for a floating hemisphere were presented in Part I. These coefficients along with the hydrostatic coefficients were then combined with the equations of motion [Eqs. (26-29)] to determine the response to wave excitation. Results were computed for the sphere assuming its center of mass to lie at its geometric center and assuming its weight to be such that it floats in the half-submerged condition.

The heave and surge response for the floating sphere is shown in Fig. 2 as a function of the period parameter $\nu=\sigma^2\bar{a}/g$. The figure shows that the sphere configuration is not highly sensitive to the finite depth effect. This is particularly true in the case of the phase shift angle of the response as presented in Fig. 3. In this figure almost no difference occurs between the results corresponding to a depth of 10.0 and 1.5 sphere radii. It is noted that the results presented in Figs. 2 and 3, corresponding to the large depth case ($h=10.0$), were compared with Kim's[8] infinite depth results, and within the resolution possible on the figures, no differences could be seen.

Figure 4 shows the heave and surge response as well as the phase shift angles for a neutrally buoyant, completely submerged sphere. The results corresponding to $h=10.0$ which represents approximately the infinite depth case are presented. The results are plotted against the dimensionless frequency parameter, $\nu=\sigma^2\bar{a}/g$, with the depth of submergence of the center of the sphere as a parameter. It is particularly in-

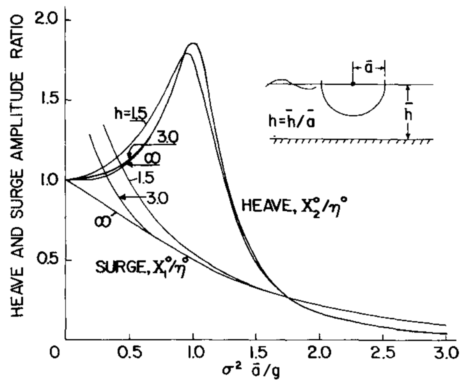


**Fig. 2 Heave and surge response of a floating sphere.**

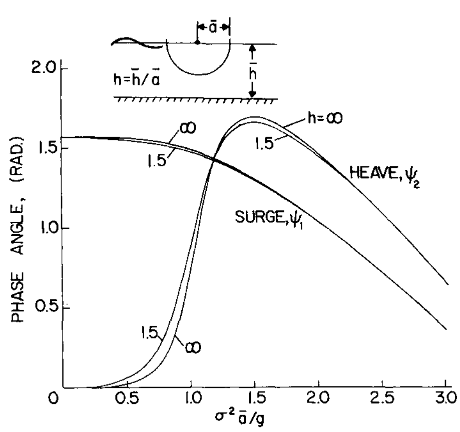


**Fig. 3 Phase shift of heave and surge response for a floating sphere.**

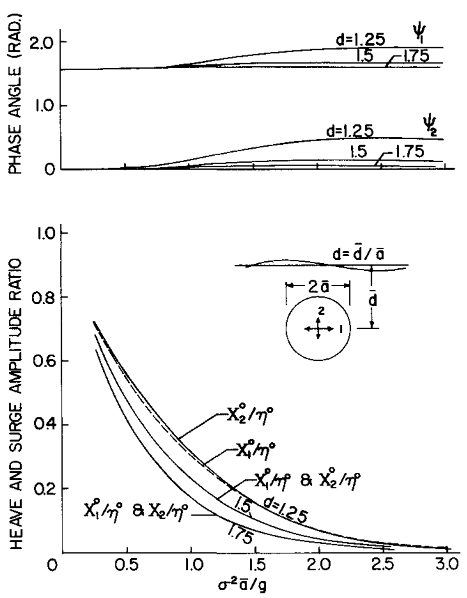


**Fig. 4 Heave and surge response of a submerged sphere in deep water.**

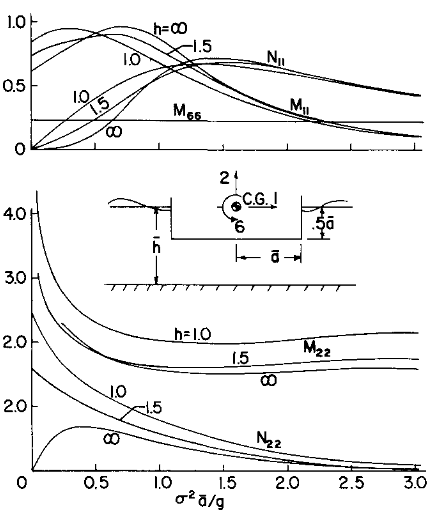


**Fig. 5 Added mass and damping coefficients for a floating vertical circular cylinder.**

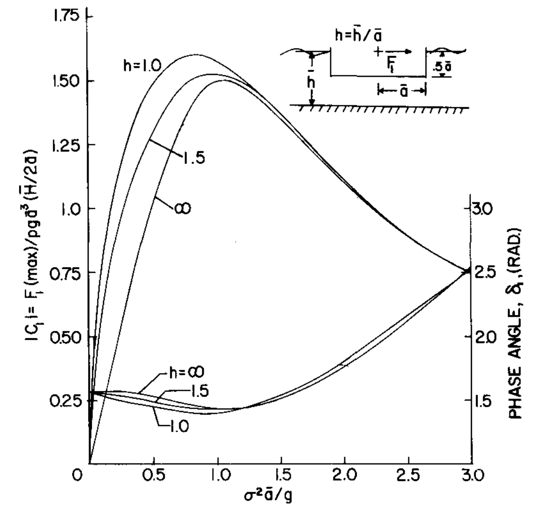


**Fig. 6 Horizontal force coefficient for a floating vertical circular cylinder.**

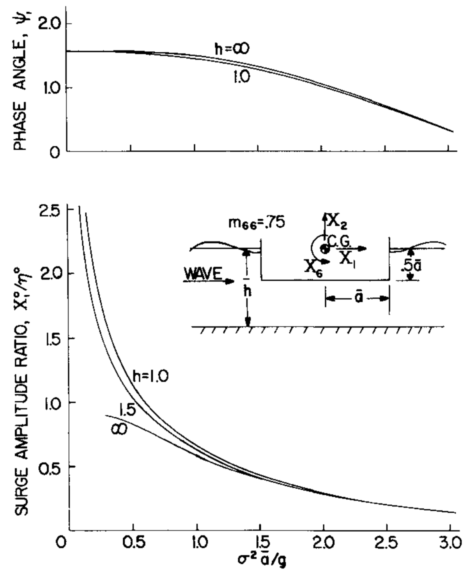


**Fig. 7 Surge response of a floating vertical circular cylinder.**

teresting to note that for this rather deep water case, little difference exists between the response amplitudes in heave and surge and the phase shift angles are approximately $\pi/2$ apart. This indicates that the sphere moves in approximately circular orbits as the fluid particles. Moreover, since no restoring force (spring constant) exists for bodies which do not pierce the free surface, no resonance condition occurs.

Gravity type oil production platforms or concrete ocean caissons generally have a large flat base. Thus, to give some idea of the expected response for such structures, a study was made for a short, circular cylinder (or circular dock). The cylinder floats upright with its axis vertical and has a diameter which is four times its draft. For purposes of presenting numerical results, its center of gravity was taken at the mean water line level and the dimensionless moment of inertia in roll was taken as $m_{66} = 0.75$.

Figure 5 shows some of the primary hydrodynamic coefficients which affect the dynamic response of the floating cylinder. These results show that the added moment of inertia in pitch, $M_{66}$, is almost independent of frequency and depth of submergence in the range of interest. The variation of $M_{66}$ with depth extended from only approximately 0.225 to 0.255 when the depth changed from $h = 10.0$ to 1.0. The added mass coefficient in heave, $M_{22}$, on the other hand, shows a rather

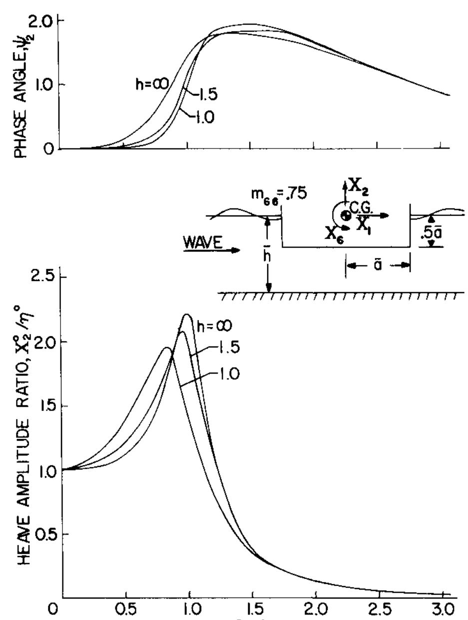


**Fig. 8 Heave response of a floating vertical circular cylinder.**

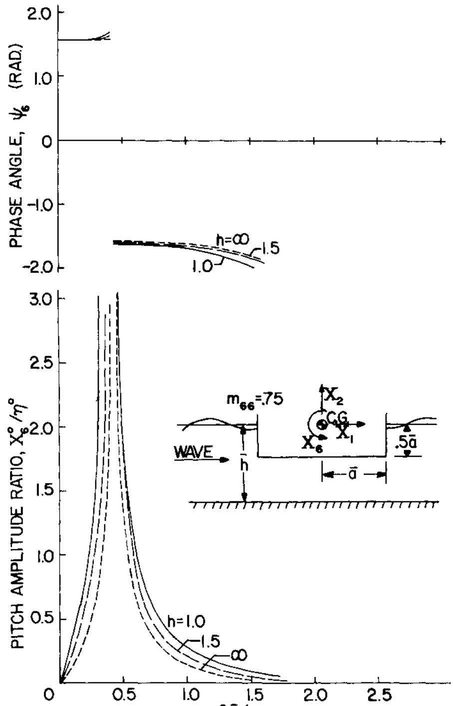


**Fig. 9 Pitch response of a floating vertical circular cylinder.**

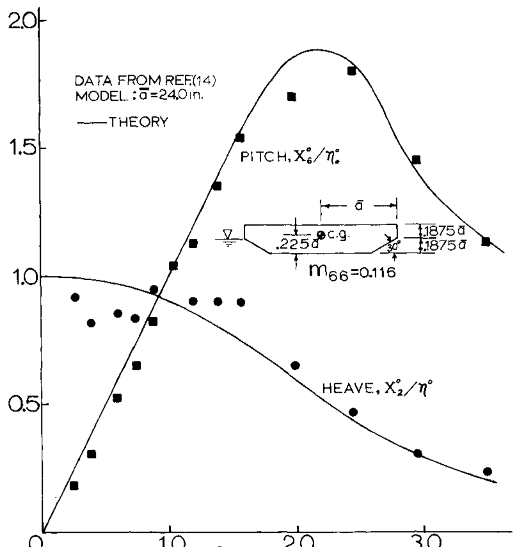


**Fig. 10 Response of a disk buoy in deep water.**

large effect of the depth. Moreover, a very large increase in this parameter may be expected when the gap between the cylinder and the bottom becomes small.

Figure 6 shows the horizontal excitation force coefficient for the vertical cylinder configuration. The results agree with those of Garrett[11] which were computed by a somewhat different method. In Fig. 7 the resulting surge response and corresponding phase shift angles for the vertical circular cylinder are presented. Figure 8 shows the corresponding heave response amplitude and phase shift angles. In the heave results, the usual resonance peak is evident but the larger damping in the shallower water cases tends to reduce the maximum response. The corresponding pitch response is shown in Fig. 9 where the effect of the very small pitch damping reflected by the large resonant peak.

Experimental results obtained by wave channel model test have been published[12] recently for the case of a "disk hull" buoy. The buoy was circular in plan and otherwise as described by the definition sketch shown in Fig. 10. In this figure, the heave and pitch results obtained from tests is compared with the present calculations and the agreement appears to be quite good.

Considerable experience with a computer program based on the present method has been accumulated and the following comments based on this experience are advanced:

One of the limitations of the method is the breakdown of the numerical results at certain "John's frequencies." These frequencies where it becomes impossible to obtain a valid solution apparently do not occur for completely submerged bodies. However, the use of the Haskind's relations and the energy balance advocated in Part I for checking the validity of the results clearly shows when the breakdown occurs so that invalid results will not be mistaken for good results.

A second limitation of the method is the complexity of the geometry of the surface of the body since the number of nodal points must be increased as the shape becomes more complex. If the shape becomes too complex the number of node points will become so large that the computer time and storage requirements for calculating the hydrodynamic coefficients will become impractical. However, the restriction to bodies having symmetry with respect to the $x-y$ plane reduces the computation time and storage requirements drastically. Further restriction from half-body to quarter-body symmetry reduces the computation time still further but has essentially no effect on the computer storage requirements. The maximum number of nodal points used by the author was 100 on the quarter body (400 total effective points).

## Conclusions

A numerical method has been developed utilizing digital computer calculations for the evaluation of wave excitation and added mass and damping coefficients of floating bodies. These results are combined, using the equations of motion, to determine the dynamic response of free floating bodies. The procedure appears to have significant application to free-floating bodies, particularly large gravity structures during the towing and sinking phase.

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